

AHP Car Selection Problem with AHP

Let us consider a simple problem. You want to buy a new car (Objective), your car selection criteria are Style, Reliability, and Fuel Economy. Right now you are considering four cars as your alternatives: Civic Coupe, Saturn Coupe, Ford Escort, and Renault Clio.

Step one: Build Hierarchy. The following diagram (Figure 1) shows your problem hierarchy.

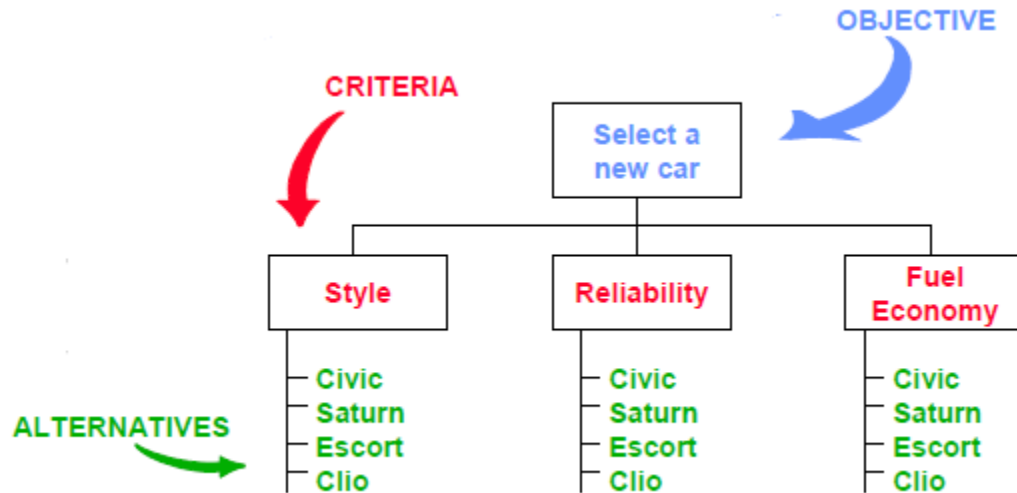


Figure 1. Problem Hierarchy.

Step Two: Establish Your Priorities

Using your own judgment, suppose that you defined the following priorities between the criteria.

- RELIABILITY is two times as important as STYLE
- STYLE is three times as important as FUEL ECONOMY
- RELIABILITY is four times as important as FULE ECONOMY
- Using the importance of each criterion relative to the others, using the scale below.

Relative Importance	Value
Equal importance/quality	1
Somewhat more important/better	3
Definitely more important/better	5
Much more important/better	7
Very much more important/better	9

Note 1:

The numbers 2, 4, 6, and 8 are half way positions between the values above.

Note 2:

The scale assumes that the ROW (first) criterion being ranked is of equal or greater importance than the COLUMN (second) criterion. If you have a pairing where the row criterion is less important than the column, use the reciprocal value (1/3, 1/5, 1/7, or 1/9).

Using the pairwise comparison, you could get the following matrix.

	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY		1/1	4/1
FUEL ECONOMY			1/1

In fact, this pairwise comparison generates the following symmetric matrix since STYLE to RELIABILITY (1/2) is converted into the RELIABILITY to STYLE comparison (2/1) – RELIABILITY is two times as important as STYLE!

	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

Step Three: Calculate the Rating

- Now let us convert the fraction form in to the decimal form for convenience.

1.0000	0.5000	3.0000
2.0000	1.0000	4.0000
0.3333	0.2500	1.0000

- Now compute the column sum for each column first

	Style	Reliability	Fuel Econ
	1.000	0.500	3.000
	2.000	1.000	4.000
	0.333	0.250	1.000
COL_SUM	3.333	1.75	8

- Divide each entry by the total of its column. For example, the cell (1, 1) = 0.3 was computed by $0.3 = 1.000/3.333$.

	Style	Reliability	Fuel Econ
Style	0.300	0.286	0.375
Reliability	0.600	0.571	0.500
Fuel Econ	0.100	0.143	0.125
COL_SUM	1.00	1.00	1.00

Note that the column sum becomes 1 since this is the weighted average at each column.

- Because the ratings are subjective, we sometimes see inconsistencies. To smooth these out, calculate the **average of each row**. This is the final weight (called the priority vector) for each criterion.

	Style	Reliability	Fuel Econ	Priority Vector (average of three)
Style	0.300	0.286	0.375	0.320
Reliability	0.600	0.571	0.500	0.557
Fuel Econ	0.100	0.143	0.125	0.123
				1.000

For example, $0.320 = (0.3 + 0.286 + 0.375)/3$. Please note that this is our last weight for each criterion.

- STYLE represents 32% of the final decision
- RELIABILITY represents 55.7% of the final decision
- FUEL ECONOMY represents 12.3% of the final decision.

See our problem structure with the criterion weight. Now we need to evaluate each alternative in terms of each criterion. However this new pairwise comparison will have the exact weight specified on the final decision.

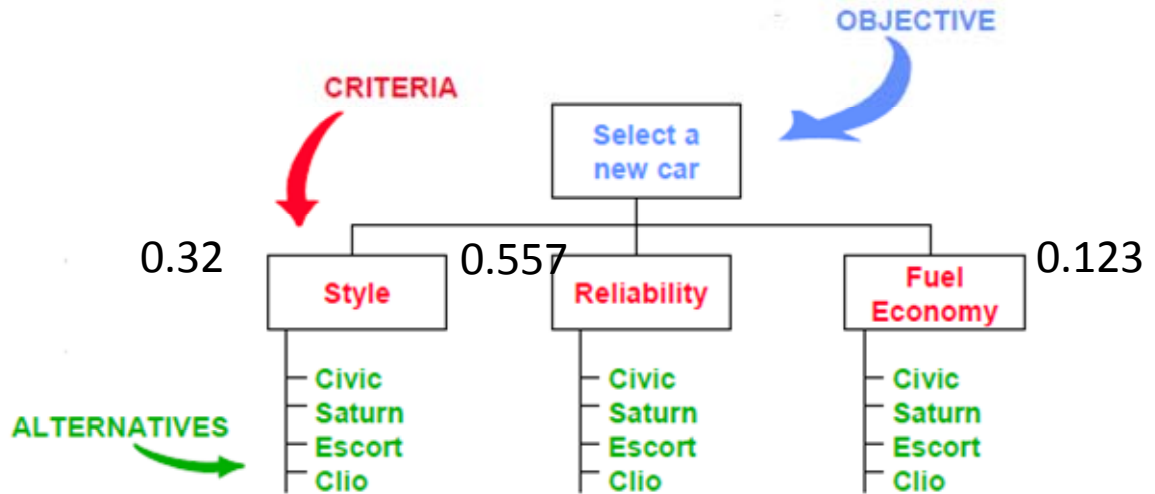


Figure 2. Overall Structure with weight on criterion

Step Four: Compare the Alternatives

- Start with the STYLE first, from the STYLE perspective, all four alternatives are evaluated through the pairwise comparison.

	STYLE			
	CIVIC	SATURN	ESCORT	CLIO
CIVIC	1/1	1/4	4/1	1/6
SATURN	4/1	1/1	4/1	1/4
ESCORT	1/4	1/4	1/1	1/5
CLIO	6/1	4/1	5/1	1/1

- Repeat the same pairwise comparison for RELIABILITY and FULE ECONOMY.

- I am going to use RELIABILITY matrix for further explanation

RELIABILITY

	CIVIC	SATURN	ESCORT	CLIO
CIVIC	1/1	2/1	5/1	1/1
SATURN	1/2	1/1	3/1	2/1
ESCORT	1/5	1/3	1/1	1/4
CLIO	1/1	1/2	4/1	1/1

- Compute the column sum and divide the each entry with the column sum as below

RELIABILITY

	CIVIC	SATURN	ESCORT	CLIO
CIVIC	1.00	2.00	5.00	1.00
SATURN	0.50	1.00	3.00	2.00
ESCORT	0.20	0.33	1.00	0.25
CLIO	1.00	0.50	4.00	1.00
SUM	2.70	3.83	13.00	4.25

RELIABILITY

	CIVIC	SATURN	ESCORT	CLIO
CIVIC	0.37	0.52	0.38	0.24
SATURN	0.19	0.26	0.23	0.47
ESCORT	0.07	0.09	0.08	0.06
CLIO	0.37	0.13	0.31	0.24
SUM	1.00	1.00	1.00	1.00

- Compute the priority vector for four alternatives in terms of the RELIABILITY.

RELIABILITY

	CIVIC	SATURN	ESCORT	CLIO	Priority Ve
CIVIC	0.37	0.52	0.38	0.24	0.38
SATURN	0.19	0.26	0.23	0.47	0.29
ESCORT	0.07	0.09	0.08	0.06	0.07
CLIO	0.37	0.13	0.31	0.24	0.26
SUM					1.00

What does this mean? This means that CIVIC represents 38% of RELIABILITY while SATURN represents 29% of RELIABILITY etc.

- Once we finish all pairwise comparison for all three criteria, we could define the problem structure as below.

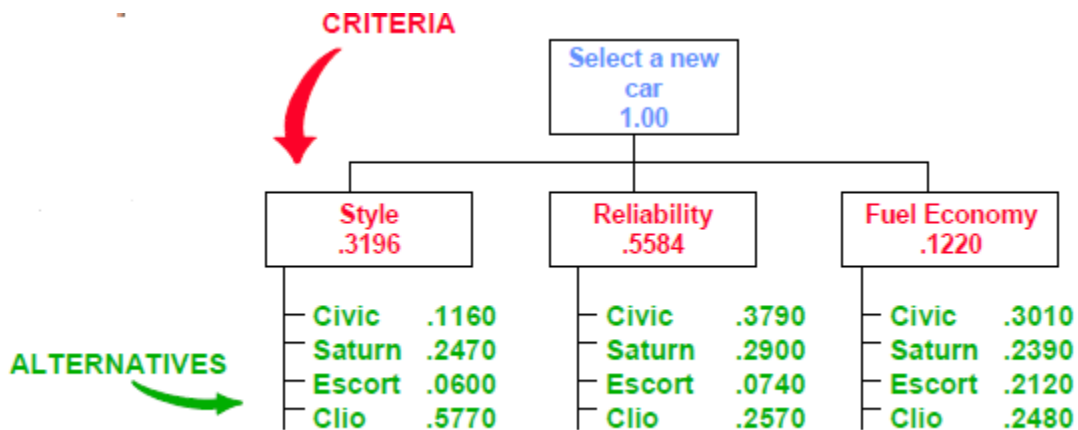


Figure 3. Overall Structure with all Weights

- Now let us evaluate all four alternatives in terms of all four criteria.

A LITTLE MORE MATRIX ALGEBRA GIVES US THE SOLUTION:

	STYLE	RELI- ABILITY	FUEL ECONOMY	CRITERIA RANKING	
CIVIC	.1160	.3790	.3010	0.3196	STYLE
SATURN	.2470	.2900	.2390	0.5584	RELIABILITY
ESCORT	.0600	.0740	.2120	0.1220	FUEL ECONOMY
CLIO	.5770	.2570	.2480		

*

I.E. FOR THE CIVIC $(.1160 * .3196) + (.3790 * .5584) + (.3010 * .1220) = .3060$

Civic	.3060	=	AND THE WINNER IS !!! THE CLIO IS THE HIGHEST RANKED CAR
Saturn	.2720		
Escort	.0940		
Clio	.3280		

This computation procedure is the sum of the weight average of all three criteria for each alternative.

For example, let us look at Clio. In figure 3, Clio's score for (STYLE, RELIABILITY, FUEL ECONOMY) was (0.5770, 0.2570, 0.2480).

These values are weighted by the criteria weight since each criterion affects the score of CLIO as follows

$$\text{Clio's score} = (0.3196)(0.5770) + (0.5584)(0.2570) + (0.1220)(0.2480) = 0.3280$$

In summary, once we decide the weight (importance) of each criterion through the pairwise comparison, each alternative is also evaluated through the pairwise comparison for each criterion. Then the weight average of all evaluation scores in terms of the criteria is computed as the final score.